Personalized Safety-focused Control by Minimizing Subjective Risk

Naren Bao¹, Dongfang Yang², Alexander Carballo^{3,4}, Ümit Özgüner² and Kazuya Takeda^{1,3,4}

Abstract—We propose a data-driven control framework for autonomous driving which combines learning-based risk assessment with personalized, safety-focused, predictive control. Different control strategies are used depending on the detected risk level of the driving situation (risky vs. non-risky). This requires a model which can understand the context of the driving situation. In addition, autonomous driving should also be able to provide various safe and comfortable driving styles customized for various users, which requires a modeling method that can capture individual driving preferences. To achieve this, we propose a novel vehicle control framework in which Model Predictive Control (MPC) is combined with a learningbased risk assessment model. Random Forest (RF) methods are trained to classify driving scenes as risky or not risky, while at the same time capturing individually preferred travel velocities. If driving scenes are classified as risky, then the Safety-focused Model Predictive Control (SMPC) system will be launched to generate control commands satisfying predetermined safety constraints, otherwise, Personalized Model Predictive Control (PMPC) is used instead to track the driver's individually preferred velocity. We demonstrate experimentally our control framework.

I. INTRODUCTION

Advanced Driving Assistance Systems (ADASs) have matured over time, so that more and more complex assistance functions are now available. However, most of the ADASs currently available are still primarily safety focused, i.e., they are pre-programmed to execute maneuvers in response to various hazardous situations, independent of the driver. Safety is without a doubt the crucial and overriding concern when designing ADAS and autonomous driving systems, but it should also be possible to provide users with a personalized driving experience. While the general concept of personalization is intuitive, the precise interpretation and goals differ between various applications and researchers [1]. The potential for personalization in ADASs was realized early on in adaptive cruise control (ACC), a driving comfort system for longitudinal control. The driver is free to choose a cruising speed among a series of pre-defined time gaps. In the work [2], ACC systems were adapted in realtime to individual drivers based on the observation of their

²D. Yang, and Ü. Özgüner are with the Electrical and Computer Engineering Department, The Ohio State University, Columbus, OH 43210, USA yang.3455@osu.edu; ozguner.1@osu.edu

⁴ A. Carballo and K. Takeda are with TierIV Inc., Nagoya University Open Innovation Center, 1-3, Meieki 1-chome, Nakamura-Ward, Nagoya, 450-6610, Japan.



Fig. 1: Framework of the proposed method of personalized safety-focused control by minimizing subjective risk as perceived by individual drivers vs. a conventional model.

driving style. Another application is personalization of the decision-making process, such as in [3], in which a data driven approach was proposed to capture the lane change decision behavior of human drivers. In [4], [5], researchers used a different approach to controller design by combining a learning-based driver model that imitates the observed driving style of the driver with Model Predictive Control to regulate the drivers desired acceleration, however why acceleration represents driving style remain unclear.

In this study, we focus on generating personalized driving control by minimizing subjective risk as perceived by experiment participants, because drivers have been found to differ in their sensitivity to risk when viewing dynamic driving scenes. This variation in risk perception influences their driving behavior, as well as their comfort level when using an ADAS. Random Forest (RF) methods are applied for driving risk assessment, because tree-based methods can be interpreted as representing a cognitive decision-making process. Moreover, we rank the importance of driving signals by seeking to minimize perceived risk, in order to capture which driving signals most influence risk perception for various individuals. Finally, Safety-focused Model Predictive Control (SMPC) is utilized to guarantee a safe following distance in risky scenarios, while in non-risky scenarios Personalized Model Predictive Control (PMPC) is used to track the drivers preferred velocity. A flowchart of our proposed method is shown in Fig. 1. Our contributions are listed as follows:

 Automated driving risk assessment: Current driving situations are classified as risky or non-risky using CAN signals and relative distances from surrounding vehicles, which can be obtained using affordable sensors. These risk assessment results can help driver as-

¹N. Bao and Κ. Takeda are with the Graduate School of Informatics, Nagoya University, Nagoya, 464-8603. bao.naren@g.sp.m.is.nagoya-u.ac.jp; Japan takeda@i.nagoya-u.ac.jp

³ A. Carballo and K. Takeda are with Institutes of Innovation for Future Society, Nagoya University, Nagoya 464-8603 Japan. alexander@g.sp.m.is.nagoya-u.ac.jp;



Fig. 2: System Architecture.

sistance systems to estimate driving states. In addition, individual differences in driving behavior as reflected by driving signals, as well as variation in driver risk perception, are investigated to better understand the preferences of drivers.

- 2) **SMPC for surrounding vehicle distance**: If risky lane changes are detected, SMPC can be used to regulate following distance to surrounding vehicles to ensure that a safe distance is maintained.
- 3) **PMPC for personalized velocity**: When non-risky lane changes are detected, PMPC is used to maintain the individually preferred velocity of the driver.

II. SYSTEM ARCHITECTURE

A detailed architecture of our proposed framework is shown Fig. 2. Our proposed framework includes offline risk assessment model described in Sec. IV, and online personalized safety-focused control of which details and experimental results are explained in Sec. V.

- Offline risk assessment model: We have trained risk assessment models including objective risk assessment model and subjective risk analysis model. In order to obtain an interpretive understanding of how individuals perceive subjective risk, RFs are applied to analyze features related to subjective risk while driving, such as driving behavior signals, as well as surrounding vehicle information. Preferred velocities are estimated for minimizing subjective risk for individuals.
- 2) Online personalized safety-focused control: At the time when the lane change decision is made, objective risk assessment model is applied to classify the current driving scene as risky or not risky. According to the risk assessment results, if a risky situation is detected then Safety-focused Model Predictive Control (SMPC) is launched to generate a safe trajectory and driving pattern. On the other hand, if a scene is classified as non-risky then individual risk-related driving signal preferences are applied to generate Personalized Model Predictive Control (PMPC).

III. OBJECTIVE RISK AND SUBJECTIVE RISK

In related studies on driving risk assessment, the terms objective risk and subjective risk have been frequently used [6]. It is important to distinguish between these two basic concepts. There is little doubt that objective risks, such as the risk of collision, are hazardous. Earlier studies of driving risk have focused on the avoidance of potential objective risks. Therefore, in our study the risk assessment module is trained to detect objective risk to be upper than threshold. However, other studies [6] rejected the idea that objective risk is a primary determinant of driver behavior, suggesting instead that drivers generally seek to avoid behavior that elicits the subjective perception of danger, and that behavior adjustments are made so as to match these estimates with a target level of acceptable risk [7]. Therefore, in this study personalized driving preferences are designed to minimize subjective risk.

- 1) Objective Risk: The objective probability of being involved in an accident.
- Subjective Risk (or Perceived Risk): The drivers own estimate of the (objective) probability of a collision. Such estimates are the output of a cognitive process.

A. Driving data

Assessments of objective and subjective risk are based on driving data, which include driver behavior signals, vehicle status and surrounding vehicle information. Driving data was collected during real-world expressway driving using multi-modal sensors mounted on an experimental vehicle [8]. Details of the collected data are described in Table I.

TABLE I:	Features	likelv	to	indicate	driving	risk.

Feature categories	No.	Features			
	1	Braking force			
Driving behavior	2	Acceleration force			
	3	Steering wheel angles			
	4	Velocity			
Vehicle states	5	Lateral acceleration			
	6	Longitudinal acceleration			

B. Surrounding vehicle information

Assessments of situational risk are also strongly influenced by the presence and activity of surrounding vehicles, therefore the environmental information used in this study includes the relative positions of surrounding vehicles in relation to the drivers own vehicle, as recorded by radar sensors. We divided the area surrounding the drivers vehicle into six areas as shown in Fig.3, and reciprocal values of the relative distances from the nearest surrounding vehicle in each area are calculated.

C. Subjective risk assessment

Data on subjective risk assessment was collected from ten experiment participants as they viewed 857 videos of lane changes (434 lane changes to the left and 423 to the right). The videos were manually extracted from dash camera



Fig. 3: Area codes for the locations of surrounding vehicles. Relative distances from the ego vehicle are measured in time steps k.

footage of real-world driving and included three seconds of video before and after each lane change. After viewing each lane change, participants were asked to report the level of risk they perceived, using a risk level score as follows: 1 = very safe, 2 = safe, 3 = neither safe nor unsafe, 4 = risky, 5 = very risky. Screenshots from a typical lane change video are shown in Fig. 4.



Fig. 4: Scenes from a typical left lane change video. Experiment participants were asked to assign risk levels to each scene (from very safe to very risky) while viewing videos of lane changes to the right and left.

D. Objective risk score definition

The objective risk level of each lane change was defined as the average of the subjective risk scores of our ten participants, which was normalized using Likert's sigma method [9]. This allows us to average out individual differences in subjective risk perception, to better approach the objective level of risk.

IV. AUTOMATED DRIVING RISK ASSESSMENT

The goal of our proposed automated risk assessment system is to be able to detect whether the current driving scene is objectively risky or not. We also investigate differences in the importance of various features of lane changes on the risk assessments of our participants.

A. Random Forest for risk classification

Random Forests (RFs) are a supervised ensemble learning method consisting of a combination of decision trees which are used as predictors. These decision trees are grown from randomized subsets (also known as bootstrap samples) [10], which use averaging to improve predictive accuracy and control over-fitting. In this study, automated driving risk assessment is formulated as a supervised classification problem. We define the driving behavior data as $\mathbf{X} \in \mathbb{R}^{D_X \times N_X}$, where D_x is the dimensionality of the data; and N_x is the amount of the data, i.e., the total number of time steps. Before the data is passed on to the RFs, a normalization

operation is performed on each element of x_t such that the columns of x_t are centered to have a mean of 0 and are scaled to have a standard deviation of 1. Objective risk $\mathbf{Y} \in \mathbb{R}^N$ is set as the target variable, where y_n is the observed risk label as [0,1], where represents (0: safe, 1: risky) for target lane change number n, and N represents the number of lane changes. If we can build many small, weak decision trees in parallel, we can then combine the trees to form a single, strong learner by averaging or taking the majority vote to determine our risk assessment result.

B. Objective risk assessment

Objective risk was set to be the target variable for risk assessment since our equations are set up as classic supervised classification problems, such that the F-measure can be used as a measurement of validity in order to measure the accuracy of our models. We counted the true positive (TP), false positive (FP) and false negative (FN) rates in ten trials and calculated the F-measures as follows:

$$F\text{-measure} = \frac{2 \times \text{Recall} \times \text{Precision}}{\text{Recall} + \text{Precision}}$$
(1)

where Precision = TP/(TP+FP) and Recall = TP/(TP+FN). We selected surrounding vehicle information and driving signals to assess objective risk. The F-measures were obtained using linear Support Vector Machine (SVM), Gaussian kernel SVM, Gaussian Process, Random Forest, Multi-Layer Perceptron (MLP) with three layers, AdaBoost and Gaussian Naive Bayes methods. The hyper-parameters of each method were tuned using grid search of 5-cross validation. The results for each F-measure computation method using our two selected risk factors are shown in Table II. Results for the top two F-measure methods are underlined for each risk factor examined separately, as well as for both risk factors examined together. In case of surrounding vehicle information, RFs scored highest, on the other hand, AdaBoost showed the best accuracy using driving signals. However, surrounding vehicle information are more influential than driving signals on risk perception [11], so that the results of this comparison showed that RFs were the most accurate method of risk assessment for use in our proposed automated risk assessment method.

TABLE II: Comparison of various F-measures for objective risk assessment using surrounding vehicle and driving signal information.

Methods	Surrounding vehicle	Driving signals	Both
SVM(linear)	0.783	0.765	0.781
SVM(RBF)	<u>0.791</u>	0.745	0.765
GaussianProcess	0.624	0.452	0.636
RF	<u>0.843</u>	<u>0.822</u>	<u>0.838</u>
MLP	0.552	0.445	0.512
AdaBoost	0.783	<u>0.862</u>	0.801
GaussianNB	0.483	0.351	0.472

C. Interpreting risk perception from driving signals

In our previous work [11], in order to investigate individual differences on subjective risk, we identified various driving-related data features which were likely to influence risk perception, and divided them into driving behavior signal features and surrounding vehicle information features. When calculating feature importance, our results were normalized to total one. The risk level rankings of ten participants in our video viewing experiment were analyzed. Our feature importance comparison results revealed that all had a strong tendency to focus mainly on frontal areas of surrounding vehicles (Area 1, Area 3, and Area 5) when assessing the riskiness of the lane change video scenes. However, this may be the result of a lack of additional information, since only frontal view video was used during our subjective risk perception data collection process as shown in Fig. 5 (a). The results of our feature importance investigation for driving signals shows variation in subjective risk during lane changes are quite influential by velocities as are shown in Fig. 5 (b). Therefore, in our study, we extract median values from velocity sequence as preferred velocities, and min and max velocities are extracted from velocity sequence for individuals to set constraints for PMPC.



Fig. 5: Feature importance analysis results for ten participants. (a) Surrounding vehicle area. (b) Driving signal.

V. MODEL PREDICTIVE CONTROL

Model predictive control (MPC) has been utilized for autonomous lane change maneuvers in various applications. MPC meets the requirement of our framework, because the desired driving speed is the primary concern in personalized safety-focused control and the absolute safety should also be considered. In our study, two types of MPCs are used to generate driving commands. The first one is safety-focused MPC (SMPC), which is used to keep a objective safe desired speed. The second one is the personalized MPC (PMPC), in which the desired speed is replaced by the personalized preferred speed obtained from the subjective risk assessment. Both MPCs have to satisfy some strict safe constraints.

A. Vehicle model

A kinematic bicycle model (see Fig. 6) is used to describe the dynamics of the ego vehicle [12], which is defined as follows:

$$\dot{x} = v \cos(\phi)$$

$$\dot{y} = v \sin(\phi)$$

$$\dot{v} = a$$

$$\dot{\phi} = \frac{v}{L} \tan(\delta)$$
(2)

where x, y are the position of the vehicle. ϕ represents the yaw angle, v is the longitudinal velocity of the vehicle, and L represents the distance from the front to rear wheels. There are two control inputs: a the longitudinal acceleration and δ the steering angle.



Fig. 6: Kinematic Bicycle Model

The above model defines the following state vector $z = [x, y, v, \phi]$. We can get a discrete-time model by using Forward Euler Discretization at time step *k* as follows:

$$z_{k+1} = f(z_k, u_k) \tag{3}$$

where $u_k = [\delta_k, a_k]$ contains the control inputs, steering angle δ and longitudinal acceleration a at time step k. If a is positive, the control input will be gas pedal operation, and if negative, the control input will be brake pedal operation, respectfully. The objective of the controller is to track the reference path (when a lane change is initialized, the reference path is switched to the target lane) by minimizing a cost function to obtain the optimal control sequence while operating within constraints.

B. Safety-focused Model Predictive Control

SMPC generates a safety maneuver by tracking a safe desired speed, which is set to 80km/h according to Japanese convention. In the meantime, a safe distance to the preceding vehicle (if there is one) should as be kept at every time step.

1) Safe Speed and Steering Constraints: In addition to physical limit on the steering angle, a more strict steering angle constraint is imposed on the SMPC, which can be expressed as:

$$\delta_{\min} \le \delta_{k|t} \le \delta_{\max} \tag{4}$$

Similarly, the speed should be constrained at every time step so that the ego vehicle only drives within a safe speed bound:

$$v_{\min,\text{safe}} \le v_{k|t} \le v_{\max,\text{safe}} \tag{5}$$

where v_{max} is the speed limit of the road.

2) Safe Distance: One primary objective of SMPC is to guarantee that the ego vehicle never collides with surrounding vehicles. This safety requirement is enforced by the following constraints on the relative distance between all possible surrounding vehicles and the ego vehicle:

$$d_{k,n|t} \ge d_{\text{safe}} \quad n \in [1, \dots, 6] \tag{6}$$

where d_{safe} is the minimum safe distance to be kept, and *n* represents the index of the surrounding vehicle areas. In the simulation, surrounding vehicle behaviors are generated by intelligent driver model (IDM) [13]. which is a crash-free car-following model configured to drive naturally.

3) Cost Function in SMPC: The cost function for SMPC is designed so that it tracks the reference trajectory associated with the desired speed:

$$J_{\text{SMPC}} = \sum_{k=t}^{t+N_c} \left[\alpha \left(z_k - z_{\text{k,ref}} \right)^2 \right], \tag{7}$$

where $z_{k,ref} = [x_{k,ref}, y_{k,ref}, v_{desired}, \phi_{k,ref}]$ is the reference path associated with the desired speed, and α is a parameter that penalizes the deviation from the reference trajectory and from the desired safe speed, respectively. SMPC focuses on maintaining safe distances to surrounding vehicles and keeping a desired speed as much as possible.

C. Personalized Model Predictive Control

The goal of PMPC is to track individually preferred velocity in the non-risky situations while satisfying safety constraints, therefore, personalized constraints and cost functions are designed as follows.

1) Safe Speed and Steering Constraints: Actuator limits follow the same constrains as with SMPC. The speed limit is enforced according to drivers individually preferred velocity limits:

$$\tilde{v}_{\min} \le v_{k|t} \le \tilde{v}_{\max} \tag{8}$$

where \tilde{v}_{min} and \tilde{v}_{max} are extracted from subjective risk analysis results.

2) Cost Function in PMPC: Similarly, the cost function:

$$J_{\text{PMPC}} = \sum_{k=t}^{t+N_c} \left[\alpha \left(z_k - z_{\text{k,ref}} \right)^2 \right], \qquad (9)$$

where $z_{k,ref} = [x_{k,ref}, y_{k,ref}, v_{individual}, \phi_{k,ref}]$, $\tilde{v}_{individual}$ is the velocity extracted from our subjective risk analysis, which is the individual preferred velocity for ten participants. α is defined the same as before.

D. Experiment setup

We validated the effectiveness of SMPC and PMPC in simulation. Parameter values of the actuator limits and safety constraints are defined in Table III. The proposed framework was evaluated in two steps. First, we compared the trajectories generated by SMPC, PMPC #1 for participant1 and #2 for participant2. They were evaluated in five lane change scenes, in which the velocity profiles were compared.

TABLE III: SMPC and PMPC controller design parameters

Parameter	Value	Parameter	Value
$\delta_{ m min}$	-15[deg]	$\delta_{ m max}$	15[deg]
v _{max,safe}	100[km/h]	d _{safe}	5[m]
v _{min,safe}	60[km/h]	v _{desired}	80[km/h]
α	0.01	β	1

E. Experimental results

1) Generated trajectory: Simulation results of the trajectories generated using SMPC, and PMPC for participant #1 are shown in Fig. 7. Surrounding vehicles were set in the same initial relative distance in both SMPC and PMPC, and running by IDM models. The trajectory generated using SMPC maintained relatively slower in the same time steps with PMPC. Trajectories generated using PMPC for participant #1 were forward than SMPC and surrounding vehicles were also driven faster.

2) Velocity profile: Our proposed framework was evaluated on switches by objective risk classification result. Velocity profiles generated for participant1 and participant2, compared with actual velocity and velocity generated by conventional MPC during five lane change scenes. The result is shown in Fig. 8. It showed that different MPCs were launched according to risk assessment result, SMPC generated lower speed, which could guarantee safety in dynamic environment, while PMPC generated preferred velocities extracted from real world data to capture individual driving patterns.

3) Subjective risk assessment: In order to validate our proposed framework, we compared average of subjective risk assessment result during lane change scenes using actual velocities, velocities from SMPC and PMPC for ten participants. The result is shown in Fig. 9. The result showed that velocities generated from SMPC increased subjective risk for participant #1, #2, #6-#10, which can be understood that safety-focused control in non-risky scenes would increase subjective risk according to individuals, however velocity generated from PMPC decrease or remain the equivalent assessment for most of the participants. Therefore, in our proposed system, PMPC can be applied to reduce the subjective risk assessment for individuals.

VI. CONCLUSION AND FUTURE WORK

In this study, we proposed a data-driven control framework for autonomous driving. Different control strategies are used depending on the driving situation (risky or non-risky). Offline, RF are trained to build objective risk assessment model, and to extract preferred velocities by minimizing subjective risk for different participants. Then, in our online personalized safety-focused system, different control strategy will be used according to the risk estimation result. In future work, we will upgrade our vehicle model to a dynamic bicycle model, which is more appropriate for high speed scenarios. Our final objective is to build a human-in-the-loop framework to generate safe, personalized control responsive to the driving situation.

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Time[s]:0.08	v_ego [km/h]:80.52				Time[s]:0.08	v_ego [km/h]:91.	91			
vehicle 2 speed [km	veh	icle 1 ed [km/h]:76.05	vehicle 0 speed [km/h]:76.	97	vehicle	2 km/h]:74.66	Vehicle 1 speed [km/h]:76.1	vehicle 0 speed [km/h]:77.0	5	
Time[s]:0.28	v_ego [km/h]:81.27		vehicle 0	176.62	Time[s]:0.28	v_ego [km/h]:92.	57	vehicle 0	76.86	
vehicl speed	e 2 [km/h]:68.95	vehicle 1 speed [km/h]:73.5			vehi spec	icle 2 ed [km/h]:69.7	vehicle 1 speed [km/h]:73.52			
Time[s]:0.9	v_ego [km/h]:80.54			vehicle 0 speed (km/h):75.58	Time[s]:0.9	v_ego [km/h]:91.	92		vehicle 0	
	vehicle 2 speed [km/h]:58	vehicle 8.57 speed [l	1 (m/h]:66.47	Speed [KII/II]. 73.30		vehicle 2 speed [km/h]:61.58 vehicle	1 km/h]:64.79	speed [km/h]:76.21	

Fig. 7: Left: Simulated SMPC. Right: Simulated PMPC #1 for participant1. The black vehicle is the ego vehicle and the blue vehicles are surrounding vehicles with the same initial positions. The red lines show the pre-defined lane change start timing, and the pink dotted lines show the trajectories.



Fig. 8: Velocity profiles generated for participant1 (red line, whose preferred velocity 91.2km/h) and participant2 (green line, whose preferred velocity 93.8km/h) compared with actual velocity and velocity generated by conventional MPC during five lane change scenes.



Fig. 9: Subjective risk assessment result. PMPC decreased or maintained subjective risk perception for most of the participants.

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